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Seismic random vibration analysis of shear beams with random structural parameters †

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Abstract

The seismic random vibration responses of shear beam structures with uncertainty are investigated. The structural mass and stiffness are considered as random variables. The excitations adopted are stationary or non-stationary random process ground accelerations in horizontal direction. Using the structural random response in frequency domain, the mean square seismic random displacements of shear beam structures are expressed as the function of random structural parameters. Computational expressions for the mean value and standard deviation of structural seismic random response are then developed by means of the random variable's functional moment method. The effects of the randomness of structural parameters on the structural random responses are demonstrated by an engineering example.

Keywords: Shear beams; Multi-story buildings; Random parameters; Random vibration; Earthquake

1. Introduction

In many engineering cases, forces acting on structures are random process excitations. Some of them can be treated as stationary random process, however, most of them such as earthquakes, blast shocks and hurricanes, should be considered as non-stationary random process. Earthquakes are most disastrous to human life and property, therefore, seismic random response of all kinds of structures and buildings has been widely investigated [1-3]. To date, the majority of modelling on seismic random response analysis of engineering structures belongs to deterministic models in which all structural parameters were regarded as deterministic parameters, especially for the civil engineering structures. In fact, most of engineering structures and buildings can be described as random structures, since they possess randomness due to variability in their material or geometric parameters or randomness resulting from the assembly/construction process and manufacturing/construction tolerances. Random structure may correspond to the randomness of a single structure such as a bridge, ship, offshore platform, antenna, building and so on or a batch of nominally identical structures such as vehicles leaving the production line [4]. Quantification of building mass depends on several factors such as materials used in construction, building dimensions, and location of non-structural elements [5]. Thus, uncertainty in building mass is the integration of uncertainties of these factors. In this context, Ellingwood et al. [6] suggested that an adequate model for the probability distribution of dead load is a normal distribution. Sources of uncertainty in structural strength and stiffness are idealization of material constitutive model, difference of member dimensions between the structural drawings and the as-built members, and imperfect knowledge of material properties. Uncertainties in structural strength and stiffness have been of major interest in a number of probabilistic studies of RC structures [7]. The randomness of dynamic response in engineering structures and buildings with uncertain parameters should not be neglected, which would bring serious effects in engineering practice. Therefore, the investigation of the problem of random structures subject to random seismic excitation is of great significance in realistic engineering applications.

The random dynamic response analysis of linear random structures has had some research attention in recent years. Wall et al. [8] and Zhao et al. [9] studied vibration of structures with random parameters to random excitations using perturbation stochastic finite element method (PSFEM) and Neumann stochastic finite element method. Li et al. [10] investigated the use of the orthogonal expansion method with the pseudo excitation method for analysing the dynamic response of structures with uncertain parameters under external

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random excitation. Ma et al. [11] solved the evolutionary earthquake response problem of an uncertain structure with bounded random parameters by a unified approach. Gao et al. [4, 12] proposed the random factor method (RFM) to investigate the effect of the uncertainty of individual structural parameter on the dynamic response of truss structures. The structural dynamic responses were developed by using the deterministic structural parameters first, and then calculated the mean value and standard deviation (statistical data) of structural responses considering the randomness of structural parameters. The current form of the RFM looks like a combination of deterministic structural dynamic analysis and probabilistic method, it seems not the real uncertain structural analysis.

In this paper, the RFM is further developed to predict the random responses of shear beams with uncertain parameters subjected to earthquake excitations. Structural parameters are considered as random variables. The dynamic characteristics, frequency response functions and mean square seismic random responses are expressed as the functions of random factors of structural parameters. Expressions for the numerical characteristics including mean value and standard deviation of the mean square seismic displacement response of shear beams are developed by the random variable's functional moment method.

2. Seismic vibration analysis of shear beam structures with random parameters

2.1 Shear beam model

A shear beam structure is shown schematically in Fig. 1(a). This simple structural model can represent various types of engineering systems such as tall buildings, dams or soil profiles. The shear beam as the model of multistorey building is widely used [13-16]. Although the shear beam seems quite simple as a building model, it is allowed to analyse complicated shear wave propagation effects which are very difficult to be explained by the traditional finite element approach commonly used in seismic engineering. Iwan [17] proposed a new format of response spectrum in 1997, the so-called drift spectrum which was based on the measures of maximum drift of a cantilever shear beam under seismic excitations. The inter-storey drift shown in Fig. 1(b) is often applied in seismic building codes since it is also a good measure of destructive effects of earthquakes on multistorey buildings. Although the shear beam is a rather simple structural model, its seismic response is in good agreement with complicated finite element analyses of building, and it is a very convenient structural model as it captures specific dynamic properties of high buildings. Chopra and Chintanapakee [18] showed that the shear beam was a good structural model and it is independent of the type of seismic excitations. Then, Sasani et al. [19] improved shear beam wave propagation model to properly account for dispersive type of damping.

The equation of motion of a uniform cantilever shear beam under horizontal seismic excitations $x(t)$ takes the following form [13]:

$$m\frac{\partial^2 u}{\partial t^2} + c\frac{\partial u}{\partial t} + c_2 k\frac{\partial^3 u}{\partial t \partial z^2} - k\frac{\partial^2 u}{\partial z^2} = -m\ddot{x}(t) \tag{1}$$

where k is the shear stiffness, m represents mass per unit length, c stands for so-called coefficient of external damping, c_2 represents interval (material) damping and $\ddot{x}(t)$ represents ground acceleration.

In Eq. (1), it is assumed that the input ground acceleration can be described by the uniformly modulated non-stationary random process as follows:

$$\ddot{x}(t) = A(t)f(t) \tag{2}$$

$A(t)$ is a given deterministic envelope (or modulation) function, and $f(t)$ is a zero-mean-valued stationary random Gaussian process. In this case, when $A(t) = 1(-\infty < t < \infty)$, the input ground acceleration $\ddot{x}(t)$ is a stationary random force and $\ddot{x}(t) = f(t)$.

Then, the equation of motion of a shear beam subjected to non-stationary random excitation can be expressed as:

$$m\frac{\partial^2 u}{\partial t^2} + c\frac{\partial u}{\partial t} + c_2 k\frac{\partial^3 u}{\partial t \partial z^2} - k\frac{\partial^2 u}{\partial z^2} = -mA(t)f(t) \tag{3}$$

Dividing both sides of Eq. (1) by m leads to the following normalized form of equation of motion for a shear beam:

$$\frac{\partial^2 u}{\partial t^2} + \frac{c\partial u}{m\partial t} + \frac{c_2 k\partial^3 u}{m\partial t \partial z^2} - \frac{k\partial^2 u}{m\partial z^2} = -A(t)f(t) \tag{4}$$

2.2 Random factor

Considering the randomness of the structural shear stiffness and mass, they can be expressed as:

$$k = \tilde{k}\cdot\bar{k} \tag{5}$$
$$m = \tilde{m}\cdot\bar{m} \tag{6}$$

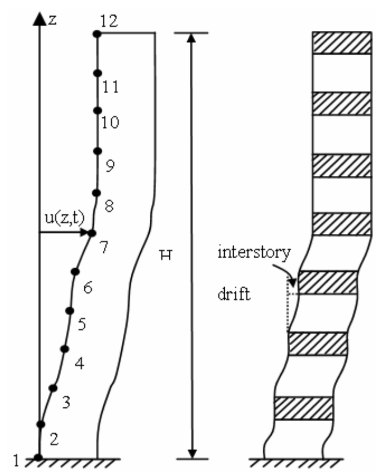

Fig. 1. Shear beam model of a multi-story building.

where $\tilde{k}$ and $\tilde{m}$ are random factors of shear stiffness k and mass m respectively, $\bar{k}$ and $\bar{m}$ are mean values of k and m respectively.

The structural shear stiffness k and its random factor $\tilde{k}$ are all random variables and obey the same probabilistic distribution. The mean value $\bar{k}$ is a deterministic value. Therefore, the randomness of k can be represented by its random factor $\tilde{k}$. Similarly, both the structural mass m and its random factor $\tilde{m}$ are random variables too. A further parameter used in the random factor method [12] is the coefficient of variation ν, defined by the ratio of the standard deviation to the mean value, that is $\nu = \sigma / \mu$. The relationships between the numerical characteristics (including mean value, standard deviation and variation coefficient) of the random structural parameters and their random factors can be expressed as:

$$\nu_{\tilde{k}} = \frac{\sigma_{\tilde{k}}}{\mu_{\tilde{k}}} = \frac{\bar{k}\sigma_{\tilde{k}}}{\bar{k}\mu_{\tilde{k}}} = \frac{\sigma_k}{\bar{k}} = \nu_k \tag{7}$$

$$\nu_{\tilde{m}} = \frac{\sigma_{\tilde{m}}}{\mu_{\tilde{m}}} = \frac{\bar{m}\sigma_{\tilde{m}}}{\bar{m}\mu_{\tilde{m}}} = \frac{\sigma_m}{\bar{m}} = \nu_m \tag{8}$$

where $\sigma_{\tilde{k}}$, $\sigma_{\tilde{m}}$, σ_k and σ_m are standard deviations of $\tilde{k}$, $\tilde{m}$, k and m respectively. $\nu_{\tilde{k}}$, $\nu_{\tilde{m}}$, ν_k and ν_m are coefficients of variations of $\tilde{k}$, $\tilde{m}$, k and m respectively.

2.3 Dynamic analysis of shear beams with random structural parameters

As shear beams deflect only in shear, their natural frequencies occur in a uniform sequence, similar to the sequence of the natural frequencies of a tall building (Fig.1). The natural frequencies ω_j and modes ϕ_j can be written in closed form solutions:

$$\omega_j = (2j-1)\frac{\pi}{2H}\sqrt{\frac{k}{m}} \tag{9}$$

$$\phi_j(z) = \sin\left(\omega_j z \Big/ \sqrt{\frac{k}{m}}\right) \tag{10}$$

where H is the height of the shear beam shown in Fig. 1.

Substituting Eqs. (5) and (6) into Eq. (9) yields:

$$\omega_j = (2j-1)\frac{\pi}{2H}\sqrt{\frac{\tilde{k}}{\tilde{m}}}\cdot\sqrt{\frac{\bar{k}}{\bar{m}}} = \sqrt{\frac{\tilde{k}}{\tilde{m}}}\cdot\bar{\omega}_j = \tilde{\omega}_j\cdot\bar{\omega}_j \tag{11}$$

where $\tilde{\omega}_j$ is the random factor the natural frequencies ω_j. $\bar{\omega}_j$ is the deterministic value of ω_j when structural parameters are deterministic. Obviously,

$$\tilde{\omega} = \sqrt{\frac{\tilde{k}}{\tilde{m}}} \tag{12}$$

$$\bar{\omega}_j = (2j-1)\frac{\pi}{2H}\sqrt{\frac{\bar{k}}{\bar{m}}} \tag{13}$$

Substituting Eqs. (5), (6) and (11) into Eq. (10) yields:

$$\phi_j(z) = \sin\left((2j-1)\frac{\pi z}{2H}\right) \tag{14}$$

Using the modal superposition method and structural vibration theory, the structural response can be expressed as:

$$u(z,t) = \sum_{j=1}^{\infty}\phi_j(z)a_j\int_0^t A(\tau)f(\tau)h_j(t-\tau)d\tau \tag{15}$$

where $h_j(t)$ are modal impulse response functions, a_j denotes modal participation factor.

Then, the power spectral density of the mean square value of structural seismic random response in frequency domain can be obtained [20, 21]:

$$S_u(t,\omega) = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j a_i\phi_j(z)\phi_i(z) \cdot A(t_1)h_j(\omega)S_f(\omega)A(t_2)h_i^*(\omega) \tag{16}$$

where $h_j(\omega)$ are the modal frequency response functions and can be expressed as:

$$h_j(\omega) = 1/(\omega_j^2 - \omega^2 + i\cdot 2\cdot\xi_j\omega_j\omega) \tag{17}$$

where $i = \sqrt{-1}$ is the complex number, ξ_j is the damping ratio of the *jth* mode and $h_j^*(\omega)$ is the complex conjugate of $h_j(\omega)$.

Substituting the Eq. (11) into Eq. (17) yields:

$$h_j(\omega) = \frac{1}{(\tilde{\omega}_j\cdot\bar{\omega}_j)^2 - \omega^2 + i\cdot 2\cdot\xi_j\cdot\tilde{\omega}_j\cdot\bar{\omega}_j\cdot\omega} \tag{18}$$

Substituting Eqs. (12) and (13) into Eq. (18) yields:

$$h_j(\omega) = \left\{\frac{\tilde{k}}{\tilde{m}}(2j-1)^2\frac{\pi^2}{4H^2}\frac{\bar{k}}{\bar{m}} - \omega^2 + i\cdot 2\cdot\xi_j\cdot\sqrt{\frac{\tilde{k}}{\tilde{m}}}\cdot(2j-1)\frac{\pi}{2H}\sqrt{\frac{\bar{k}}{\bar{m}}}\cdot\omega\right\}^{-1} \tag{19}$$

After integrating $S_u(t,\omega)$ within the frequency domain, the mean square displacements along the height Z of the shear beam equals:

$$\psi_{uZ}^2 = \int_{-\infty}^{\infty}S_u(t,\omega)d\omega = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j a_i\phi_j(z)\phi_i(z) \cdot\int_{-\infty}^{\infty}A(t_1)h_j(\omega)S_f(\omega)A(t_2)h_i^*(\omega)d\omega \tag{20}$$

3. Numerical characteristics of seismic response

Obviously, when the shear stiffness and mass are random variables, the natural frequencies ω_j is also a random variable because it is the function of them as shown in Eq. (11). However, the change of structural stiffness and mass does not affect the modal shapes as expected from Eq. (14). Therefore, by means of the algebra synthesis method which can be found in many books (also see Appendix A), expressions for mean value μ_{ω_j}, standard deviation σ_{ω_j} and variation coefficient v_{ω_j} of the *jth* natural frequency ω_j can be obtained in terms of the variation coefficients $v_{\tilde{k}}$ and $v_{\tilde{m}}$ of the random factors $\tilde{k}$ and $\tilde{m}$:

$$\mu_{\omega_j} = (2j-1)\frac{\pi}{2H}\cdot\sqrt{\frac{\mu_k}{\mu_m}}\cdot\sqrt{\frac{\mu_{\tilde{k}}}{\mu_{\tilde{m}}}} \cdot\sqrt{\sqrt{\left(1+\frac{v_{\tilde{m}}^2\cdot\mu_{\tilde{m}}^2}{\mu_{\tilde{k}}^2}\right)^2-\frac{1}{2}\left(v_{\tilde{k}}^2+v_m^2\right)}} \tag{21}$$

$$\sigma_{\omega_j} = (2j-1)\frac{\pi}{2H}\cdot\sqrt{\frac{\mu_k}{\mu_m}}\cdot\sqrt{\frac{\mu_{\tilde{k}}}{\mu_{\tilde{m}}}}\cdot\left\{\left(1+\frac{v_{\tilde{m}}^2\cdot\mu_{\tilde{m}}^2}{\mu_{\tilde{k}}^2}\right) \left(\left(1+\frac{v_{\tilde{m}}^2\cdot\mu_{\tilde{m}}^2}{\mu_{\tilde{k}}^2}\right)^2-\frac{1}{2}\left(v_{\tilde{k}}^2+v_m^2\right)\right)^{1/2}\right\}^{1/2} \tag{22}$$

$$v_{\omega_j} = \frac{\sigma_{\omega_j}}{\mu_{\omega_j}} \tag{23}$$

where $\mu_{\tilde{K}} = 1$ and $\mu_{\tilde{m}} = 1$, $v_{\tilde{k}} = v_m$ and $v_{\tilde{m}} = v_m$.

From Eq. (19), it can be easily seen that the frequency response functions are also a random variables if the shear stiffness and mass are variables. By mean of the random variable's functional moment method [20], the expressions for mean value $\mu_{h_j(\omega)}$ and standard deviation $\sigma_{h_j(\omega)}$ of the frequency response function can be obtained:

$$\mu_{h_j(\omega)} = 1/FRF \tag{24}$$

$$FRF = \left\{\frac{\mu_{\tilde{k}}}{\mu_{\tilde{m}}}(2j-1)^2\frac{\pi^2}{4H^2}\frac{\mu_k}{\mu_m}-\omega^2 +i\cdot 2\cdot\xi_j\cdot\sqrt{\frac{\mu_{\tilde{k}}}{\mu_{\tilde{m}}}}\cdot(2j-1)\frac{\pi}{2H}\sqrt{\frac{\mu_k}{\mu_m}}\cdot\omega\right\} \tag{25}$$

$$\sigma_{h_j(\omega)} = \frac{1}{(FRF)^2}\left\{\left[\left(\frac{\mu_{\tilde{k}}}{\mu_{\tilde{m}}^2}(2j-1)^2\frac{\pi^2}{4H^2}\frac{\mu_k}{\mu_m} +i\cdot\xi_j\sqrt{\frac{\mu_{\tilde{k}}}{\mu_{\tilde{m}}^3}}(2j-1)\frac{\pi}{2H}\sqrt{\frac{\mu_k}{\mu_m}}\omega\right)v_{\tilde{m}}\mu_{\tilde{m}}\right]^2 +\left[\left(\frac{1}{\mu_{\tilde{m}}}(2j-1)^2\frac{\pi^2}{4H^2}\frac{\mu_k}{\mu_m}-i\cdot\xi_j\frac{1}{\sqrt{\mu_{\tilde{k}}\cdot\mu_{\tilde{m}}}} (2j-1)\frac{\pi}{2H}\sqrt{\frac{\mu_k}{\mu_m}}\omega\right)v_{\tilde{k}}\mu_{\tilde{k}}\right]^2\right\}^{1/2} \tag{26}$$

Similarly, the expressions for mean value $\mu_{\psi_{uZ}^2}$ and standard deviation $\sigma_{\psi_{uZ}^2}$ of mean square displacement can also be obtained by mean of the random variable's functional moment method:

$$\mu_{\psi_{uZ}^2} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j\,a_i\,\phi_j(z)\phi_i(z) \cdot\int_{-\infty}^{\infty}A(t_1)\mu_{h_j(\omega)}S_f(\omega)A(t_2)\mu_{h_i^*(\omega)}d\omega \tag{27}$$

$$\sigma_{\psi_{uZ}^2} = \left\{\left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j\,a_i\,\phi_j(z)\phi_i(z)\cdot\sigma_{h_j(\omega)} \cdot\int_{-\infty}^{\infty}A(t_1)S_f(\omega)A(t_2)\mu_{h_i^*(\omega)}d\omega\right)^2 +\left(\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_j\,a_i\,\phi_j(z)\phi_i(z)\cdot\sigma_{h_i^*(\omega)} \cdot\int_{-\infty}^{\infty}A(t_1)\mu_{h_j(\omega)}S_f(\omega)A(t_2)\,d\omega\right)^2\right\}^{1/2} \tag{28}$$

Then the variation coefficient of mean square displacement along the height Z of the shear beam $v_{\psi_{uZ}^2}$ can obtained as:

$$v_{\psi_{uZ}^2} = \frac{\sigma_{\psi_{uZ}^2}}{\mu_{\psi_{uZ}^2}} \tag{29}$$

4. Numerical example

The shear beam shown in Fig. 1 is used as an example to investigate the effect of randomness of structural parameters on multi-storey buildings. The height of shear beam is $H = 35m$. The mean value of the structural shear stiffness and mass are $\mu_k = 2.13\times10^9 kgm/s^2$ and $\mu_m = 9.8\times10^5 kg/m$ respectively. The cantilever beam with such parameters can roughly represent an 11-story prefabricated building which is usually built with the same cross-section along its height.

In the following calculations, the values of all damping ratios are taken as $\xi_j = 0.01$. It is a violation of the modal decoupling requirements but keeping constant damping makes it easier to identify the contributions of various modes regardless of the applied damping hypothesis. Such assumption is also recommended in the recent paper on shear beam seismic vibrations [19].

In order to illustrate the effect of randomness of random variables k and m on the natural frequencies and seismic random response, different values for the variation coefficients of the random variables are examined. The maximum values of variation coefficients of random variables are usually taken as 0.3 due to the 3σ law. In this study, different groups of the values of variation coefficients are taken into account only to illustrate the presented method.

4.1 Natural frequencies

Mean value, standard deviation and variation coefficient of the first six natural frequencies are given in Tables 1, 2 and 3

Table 1. Mean value.

	μ_{ω_1}	μ_{ω_2}	μ_{ω_3}
$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	2.0711	6.2132	10.3554
$\nu_{\tilde{k}} = 0.2$	2.0818	6.2453	10.4089
$\nu_{\tilde{m}} = 0.2$	2.1238	6.3715	10.6191
$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.3$	2.1419	6.4256	10.7093
$\nu_{\tilde{k}} = 0.3$	2.0684	6.2051	10.3419
$\nu_{\tilde{m}} = 0.3$	2.1635	6.4904	10.8173

Table 2. Standard deviation.

	σ_{ω_1}	σ_{ω_2}	μ_{ω_3}
$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	0.2915	0.8745	1.4576
$\nu_{\tilde{k}} = 0.2$	0.2098	0.6293	1.0488
$\nu_{\tilde{m}} = 0.2$	0.2056	0.6169	1.0282
$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.3$	0.4293	1.2879	2.1465
$\nu_{\tilde{k}} = 0.3$	0.3156	0.9469	1.5782
$\nu_{\tilde{m}} = 0.3$	0.3021	0.9062	1.5103

Table 3. Variation coefficient (* MCSM).

	ν_{ω_1}	ν_{ω_2}	ν_{ω_3}
$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	0.1408	0.1408	0.1408
$^*\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	* 0.1418	* 0.1419	* 0.1421
$\nu_{\tilde{k}} = 0.2$	0.1008	0.1008	0.1008
$\nu_{\tilde{m}} = 0.2$	0.0968	0.0968	0.0968
$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.3$	0.2004	0.2004	0.2004
$\nu_{\tilde{k}} = 0.3$	0.1526	0.1526	0.1526
$\nu_{\tilde{m}} = 0.3$	0.1396	0.1396	0.1396

in terms of different groups of the values of variation coefficients of random structural stiffness and mass respectively. In addition, in order to verify the effectiveness of the method presented in this paper, variation coefficient of the first six natural frequencies obtained by Monte-Carlo simulation method (MCSM) are also given in Table 3 when $\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$, in which 10000 simulations are used.

Table 3 shows that the results obtained by the method presented in this paper are in good agreement with results obtained from the Monte-Carlo simulation method. It is observed that the uncertainties of structural shear stiffness and mass produce the same effect on the mean value of natural frequencies of shear beams. From Tables 2 and 3, it can be seen that the uncertainty of structural stiffness will cause greater effect on the randomness of natural frequencies. The variation coefficients of all natural frequencies are same as expected from Eq. (23), because the simple shear beam mode is adopted in this paper. However, the standard deviations of higher order natural frequencies are bigger as their mean values are bigger and higher order modes are more sensitive to the uncertainties of structural parameters. In addition, along with the increase of the variation coefficients of structural parameters, the uncertainty of structural natural frequencies will also increase.

Table 4. Mean value (unit: m^2).

	$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	$\nu_{\tilde{k}} = 0.2$	$\nu_{\tilde{m}} = 0.2$
$\mu_{\psi_{u12}^2}$	0.0432	0.0432	0.0432
$\mu_{\psi_{u11}^2}$	0.0412	0.0412	0.0412
$\mu_{\psi_{u10}^2}$	0.0380	0.0380	0.0380
$\mu_{\psi_{u9}^2}$	0.0339	0.0339	0.0339
$\mu_{\psi_{u8}^2}$	0.0292	0.0292	0.0292
$\mu_{\psi_{u7}^2}$	0.0242	0.0242	0.0242
$\mu_{\psi_{u6}^2}$	0.0190	0.0190	0.0190
$\mu_{\psi_{u5}^2}$	0.0140	0.0140	0.0140
$\mu_{\psi_{u4}^2}$	0.0094	0.0094	0.0094
$\mu_{\psi_{u3}^2}$	0.0053	0.0053	0.0053
$\mu_{\psi_{u2}^2}$	0.0021	0.0021	0.0021
$\mu_{\psi_{u1}^2}$	0.0002	0.0002	0.0002

Table 5. Standard deviation (unit: m^2).

	$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	$\nu_{\tilde{k}} = 0.2$	$\nu_{\tilde{m}} = 0.2$
$\sigma_{\psi_{u12}^2}$	0.0065	0.0044	0.0047
$\sigma_{\psi_{u11}^2}$	0.0063	0.0043	0.0046
$\sigma_{\psi_{u10}^2}$	0.0059	0.0040	0.0043
$\sigma_{\psi_{u9}^2}$	0.0053	0.0036	0.0038
$\sigma_{\psi_{u8}^2}$	0.0045	0.0031	0.0033
$\sigma_{\psi_{u7}^2}$	0.0037	0.0025	0.0027
$\sigma_{\psi_{u6}^2}$	0.0028	0.0019	0.0021
$\sigma_{\psi_{u5}^2}$	0.0020	0.0014	0.0014
$\sigma_{\psi_{u4}^2}$	0.0012	0.0008	0.0009
$\sigma_{\psi_{u3}^2}$	0.0020	0.0004	0.0005
$\sigma_{\psi_{u2}^2}$	0.0002	0.0001	0.0002
$\sigma_{\psi_{u1}^2}$	0.0000	0.0000	0.0000

4.2 Stationary and non-stationary seismic random response

For the stationary random ground acceleration, let $A(t) = 1$ and the power spectral density of $f(t)$ is described by the following Kanai-Tajimi filtered-white-noise spectrum [20]:

$$S_f(\omega) = \frac{1 + 4\varsigma_g^2 (\omega/\omega_g)^2}{\left(1 - (\omega/\omega_g)^2\right)^2 + 4\varsigma_g^2 (\omega/\omega_g)^2} S_0 \tag{30}$$

where $\omega_g = 16.5 rad/s$, $\varsigma_g = 0.7$, $S_0 = 0.0156 m^2/s^3$.

Computational results of the mean value $\mu_{\psi_{uj}^2}$ $(j = 1,2,...,12)$ of the mean square value of the displacement response ψ_{uj}^2 $(j = 1,2,...,12)$ of each node (level) of building are given in Table 4. Tables 5 and 6 present the computational results of the standard deviation $\sigma_{\psi_{uj}^2}$ $(j = 1,2,...,12)$ and the variation coefficient $\nu_{\psi_{uj}^2}$ $(j = 1,2,...,12)$ of the mean square value of structural displacement response.

Table 6. Variation coefficient.

	$\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$	$\nu_{\tilde{k}} = 0.2$	$\nu_{\tilde{m}} = 0.2$
$\nu_{\psi_u^2 12}$	0.1395	0.0956	0.1017
$\nu_{\psi_u^2 11}$	0.1408	0.0964	0.1027
$\nu_{\psi_u^2 10}$	0.1427	0.0977	0.1041
$\nu_{\psi_u^2 9}$	0.1438	0.0985	0.1049
$\nu_{\psi_u^2 8}$	0.1440	0.0986	0.1050
$\nu_{\psi_u^2 7}$	0.1431	0.0981	0.1043
$\nu_{\psi_u^2 6}$	0.1409	0.0966	0.1027
$\nu_{\psi_u^2 5}$	0.1370	0.0940	0.0997
$\nu_{\psi_u^2 4}$	0.1309	0.0899	0.0952
$\nu_{\psi_u^2 3}$	0.1221	0.0840	0.0887
$\nu_{\psi_u^2 2}$	0.1113	0.0768	0.0807
$\nu_{\psi_u^2 1}$	0.1030	0.0711	0.0745

From Table 4, it can be seen that the mean values of the mean square seismic random displacements increase along the height of the structure, that is, higher stories have bigger seismic responses. Similarly, standard deviations of higher stories are also bigger as shown in Table 5. Generally speaking, the uncertainties of structural parameters will produce grater effect on the structural responses of higher levels, but it is hard to say they will produce the biggest effect on the highest level because its corresponding variation coefficient may not be the biggest one. For example, the variation coefficient of level 12 $\nu_{\psi_u^2 12}$ = 0.1395 which is less than the variation coefficient of level 11 $\nu_{\psi_u^2 11}$ =0.1408 in Table 6, although the standard deviation $\sigma_{\psi_u^2 12}$ =0.0065 m^2 is greater than $\sigma_{\psi_u^2 11}$ =0.0063 m^2 in Table 5 when $\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$.

For the non-stationary random ground excitation, the modified version of the Kanai-Tajimi model suggested in Refs [21] is adopted:

$$S_f(\omega) = \frac{\omega_g^4 + 4\omega^2\omega_g^2\varsigma_g^2}{\left(\omega^2 - \omega_g^2\right)^2 + 4\omega^2\omega_g^2\varsigma_g^2} \cdot \frac{\omega^4}{\left(\omega^2 - \omega_f^2\right)^2 + 4\omega^2\omega_g^2\varsigma_g^2} S_0 \tag{31}$$

where S_0 is the ordinate of the power spectral density of the bedrock acceleration. ω_g and ς_g are the natural frequency and critical damping ratio of soil layer, ω_f and ς_f are meters of a second filter which is introduced to assure a finite power for the ground displacement. In this example, $\omega_g = 15.0 rad/s$, $\varsigma_g = 0.6$, $S_0 = 1.0 m^2/s^3$, $\omega_f = 1.5 rad/s$, and the following envelop function of the input acceleration [22] is considered:

$$A(t) = \exp(-0.13t) - \exp(-0.45t) \tag{32}$$

For non-stationary seismic responses, curves of the mean value, mean value plus standard deviation ($\mu + \sigma$) and mean value minus standard deviation ($\mu - \sigma$) of the mean square value of displacement response ψ_{u7}^2 of the node (level) 7 of shear beam are shown in Figs. 2 to 4.

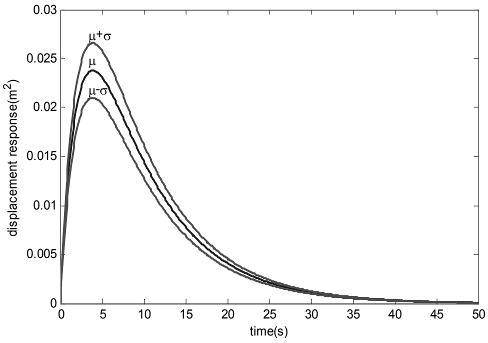

Fig. 2. Mean square displacement ($\nu_{\tilde{k}} = \nu_{\tilde{m}} = 0.2$).

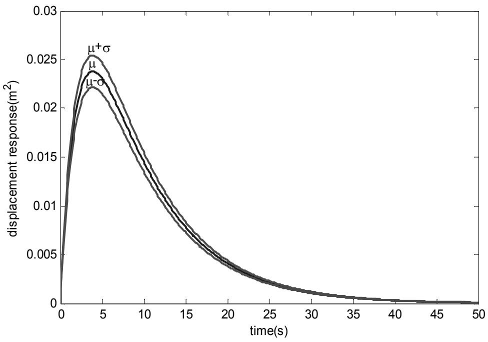

Fig. 3. Mean square displacement ($\nu_{\tilde{k}} = 0.2$).

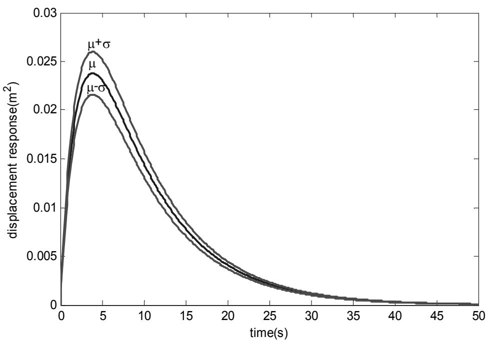

Fig. 4. Mean square displacement ($\nu_{\tilde{m}} = 0.2$).

From Tables 4-6 and Figures 2-4, it can be obtained that: (1) the effect of the randomness of stiffness and mass on the mean square value of structural non-stationary and stationary seismic random displacement are different. (2) The randomness of mass will produce greater effect on random seismic dynamic displacement of shear beam. (3) The change range of structural random response is much bigger if the randomness of stiffness and mass are considered simultaneously.

5. Conclusion

The objective of this study is to predict the random responses of shear beam structures with uncertain parameters under the stationary and non-stationary random earthquake excitations. Using the RFM, a random structural parameter is expressed as the product of its mean value and random factor. The randomness of a structural parameter can be represented by its random factor. Mathematic expressions of the mean value, standard deviation and variation coefficient for the structural natural frequencies, mean square seismic responses of shear beams have been developed. The dynamic responses of shear beams with uncertainty under random seismic excitations are obtained expediently.

The effects of the uncertainties of structural parameters on the seismic response of different stories are different. In general, higher stories will have bigger change ranges in their seismic displacements if structural parameters are variables. Therefore, structural designer should be more careful when they design tall buildings.

Acknowledgements

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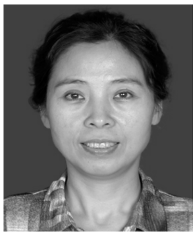
Appendix A. Algebra synthesis method

Table A1. Numerical characteristics analysis using the algebra synthesis method.

Z=f (X,Y)	μ_Z	σ_Z
Z=a	a	
Z=aX	$a\mu_X$	$a\sigma_X$

Z=aX+b	aX+b	$a\sigma_X$
Z=aX-b	aX-b	$a\sigma_X$
Z=X+Y	$\mu_X + \mu_Y$	$(\sigma_X^2 + \sigma_Y^2 + 2c_{XY}\sigma_X\sigma_Y)^{1/2}$
Z=X-Y	$\mu_X - \mu_Y$	$(\sigma_X^2 + \sigma_Y^2 - 2c_{XY}\sigma_X\sigma_Y)^{1/2}$
Z=XY	$\mu_X\mu_Y + c_{XY}\sigma_X\sigma_Y$	$(\mu_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2 + \sigma_X^2\sigma_Y^2$ $+2c_{XY}\mu_X\mu_Y\sigma_X\sigma_Y$ $+c_{XY}^2\sigma_X^2\sigma_Y^2)^{1/2}$
Z=X/Y	$\frac{\mu_X}{\mu_Y}[1+\frac{\sigma_Y}{\mu_Y}$ $(\frac{\sigma_Y}{\mu_Y} - c_{XY}\frac{\sigma_X}{\mu_X})]$	$[\frac{\mu_X^2}{\mu_Y^2}(\frac{\sigma_X^2}{\mu_X^2} + \frac{\sigma_Y^2}{\mu_Y^2}$ $-2c_{XY}\frac{\sigma_X\sigma_Y}{\mu_X\mu_Y})]^{1/2}$
Z=X^2	$\mu_X^2 + \sigma_X^2$	$(4\mu_X^2\sigma_X^2 + 2\sigma_X^4)^{1/2}$ $\approx 2\mu_X\sigma_X$
Z=X^3	$\mu_X^3 + 3\mu_X\sigma_X^2$	$(3\sigma_X^6 + 8\mu_X^2\sigma_X^4$ $+5\mu_X^4\sigma_X^2)^{1/2} \approx 3\mu_X^2\sigma_X$
$Z = X^n$	$\approx \mu_X^n$	$\approx \lvert n \rvert \mu_X^{n-1}\sigma_X$
$Z = X^{1/2}$	$(\sqrt{(\mu_X^2 - \frac{1}{2}\sigma_X^2)})^{1/2}$	$(\mu_X - \sqrt{(\mu_X^2 - \frac{1}{2}\sigma_X^2)})^{1/2}$
Z=X^2+Y^2	$(\mu_X^2 + \mu_Y^2)^{1/2}$ $+\frac{\mu_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2}{2\sqrt{(\mu_X^2 + \mu_Y^2)^3}}$	$(\frac{\mu_X^2\sigma_Y^2 + \mu_Y^2\sigma_X^2}{\mu_X^2 + \mu_Y^2})^{1/2}$

Suppose that X and Y are normal (Gaussian) random variables, the mean value μ_Z and standard deviation σ_Z of random variable $Z = f(X,Y)$ are given in Table A1. In this table, α and β are constants, μ_X and μ_Y are the mean value of X and Y respectively, σ_X and σ_Y are the standard deviation of X and Y respectively, and c_{XY} is the correlation coefficient of X and Y. Two extreme situations exist for the value of the correlation coefficient. If the variables X and Y are independent, then $c_{XY} = 0$. If X is completely positively correlated with Y, then $c_{XY} = 1$. If X and Y are not normal random variables, they should be transformed as normal random variables before using the equations given in Table A1.



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